

Generalized noise terms for the quantized fluctuational electrodynamics

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The quantization of optical fields in vacuum has been known for decades, but extending the field quantization to lossy and dispersive media in nonequilibrium conditions has proven to be complicated due to the position-dependent electric and magnetic responses of the media. In fact, consistent position-dependent quantum models for the photon number in resonant structures have only been formulated very recently and only for dielectric media. Here we present a general position-dependent quantized fluctuational electrodynamics (QFED) formalism that extends the consistent field quantization to describe the photon number also in the presence of magnetic field-matter interactions. It is shown that the magnetic fluctuations provide an additional degree of freedom in media where the magnetic coupling to the field is prominent. Therefore, the field quantization requires an additional independent noise operator that is commuting with the conventional bosonic noise operator describing the polarization current fluctuations in dielectric media. In addition to allowing the detailed description of field fluctuations, our methods provide practical tools for modeling optical energy transfer and the formation of thermal balance in general dielectric and magnetic nanodevices. We use the QFED to investigate the magnetic properties of microcavity systems to demonstrate an example geometry in which it is possible to probe fields arising from the electric and magnetic source terms. We show that, as a consequence of the magnetic Purcell effect, the tuning of the position of an emitter layer placed inside a vacuum cavity can make the emissivity of a magnetic emitter to exceed the emissivity of a corresponding electric emitter.

I. INTRODUCTION

Better understanding of optical phenomena and nanoscale energy transfer has enabled advances in optical technologies, e.g., in nanoplasmonics [1–4], near-field microscopy [5, 6], thin-film light-emitting diodes [7, 8], photonic crystals [9, 10], and metamaterials [11, 12]. These advances are strongly influenced by the availability of simple and transparent theoretical tools that allow in-depth understanding of the pertinent phenomena in sufficiently simple form. Formulating a simple and sufficiently detailed description of the quantum aspects of energy transfer in lossy resonant structures, however, has been particularly challenging due to several phenomena, such as wave-particle dualism, intertwined electric and magnetic fields, and field-matter interactions [13, 14], affecting the energy transfer.

We have recently introduced quantized fluctuational electrodynamics (QFED) formalism [15–18] for the description of field-matter interactions and the formation of thermal balance in nonequilibrium conditions in dielectric media. Using the QFED approach it has finally become possible to formulate the canonical commutation relations preserving ladder and photon-number operators for the total electromagnetic field also in resonant structures [15–17], thus resolving the previously reported anomalies in their commutation relations [19–23]. Here we present a more general QFED formalism that also accounts for the interactions arising due to magnetic effects. The need to formulate the unified theory emerges

from the present observation that the single electric noise operator picture is insufficient to correctly describe the field-matter interactions in the general case. Instead, the use of two commuting bosonic noise source operators is needed to formulate the general model. By using two separate operators it becomes straightforward to develop the QFED model starting from the macroscopic Maxwell's equations and the polarization and magnetization related material responses.

After deriving the generalized QFED formalism, we apply it to show that, due to the magnetic Purcell effect, we can tune the position of the emitter in a vacuum cavity such that the emissivity of a magnetic emitter significantly exceeds the emissivity of a corresponding electric emitter. This is a consequence of different position dependences of the electric and magnetic local densities of states (LDOSs). The differences in emissivities are expected to be experimentally observable by detecting the output radiation with a photodetector or an antenna.

This manuscript is organized as follows: The theory is presented in Sec. II. It covers the introduction of the quantized equations, representation of the Green's functions and the related noise source operators. We also briefly review the theory of field fluctuations, photon numbers, Poynting vector, and absorption and emission operators while presenting the new generalized forms of the densities of states. In Sec. III, we investigate the physical implications of the concepts by applying the methods to study the emissivity of electric and magnetic emitters placed inside the vacuum cavity and demonstrate the different characteristics of the two fundamen-

tally different source terms of our theory. Finally, conclusions are drawn in Sec. IV.

II. FIELD QUANTIZATION

A. Quantized equations

Maxwell's equations relate the electric field strength \mathbf{E} , the magnetic field strength \mathbf{H} , the electric flux density \mathbf{D} , and the magnetic flux density \mathbf{B} to the free electric charge ρ_f and current \mathbf{J}_f densities [24]. In the frequency domain, the equations for positive frequencies read

$$\nabla \cdot \mathbf{D} = \rho_f, \quad (1)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2)$$

$$\nabla \times \mathbf{E} = i\omega \mathbf{B} = i\omega \mu_0 (\mu \mathbf{H} + \delta \mathbf{M}), \quad (3)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f - i\omega \mathbf{D} = \mathbf{J}_f - i\omega (\varepsilon_0 \varepsilon \mathbf{E} + \delta \mathbf{P}), \quad (4)$$

where we have related the fields and field densities in Eqs. (3) and (4) using the constitutive relations $\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} + \delta \mathbf{P}$ and $\mathbf{B} = \mu_0 (\mu \mathbf{H} + \delta \mathbf{M})$, where ε_0 and μ_0 are the permittivity and permeability of vacuum, $\varepsilon = \varepsilon_r + i\varepsilon_i$ and $\mu = \mu_r + i\mu_i$ are the relative permittivity and permeability of the medium with real and imaginary parts denoted by subscripts r and i, and the polarization and magnetization fields $\delta \mathbf{P}$ and $\delta \mathbf{M}$ denote the polarization and magnetization that are not linearly proportional to the respective field strengths [25]. In the context of the present work, $\delta \mathbf{P}$ and $\delta \mathbf{M}$ describe small noise related parts in the linear polarization and magnetization fields as customary in the the classical fluctuational electrodynamics [26].

From the Maxwell's equations in Eqs. (1)–(4) it follows that the electric and the magnetic fields obey the well-known equations

$$\nabla \times \left(\frac{\nabla \times \mathbf{E}}{\mu_0 \mu} \right) - \omega^2 \varepsilon_0 \varepsilon \mathbf{E} = i\omega \mathbf{J}_e - \nabla \times \left(\frac{\mathbf{J}_m}{\mu_0 \mu} \right), \quad (5)$$

$$\nabla \times \left(\frac{\nabla \times \mathbf{H}}{\varepsilon_0 \varepsilon} \right) - \omega^2 \mu_0 \mu \mathbf{H} = i\omega \mathbf{J}_m + \nabla \times \left(\frac{\mathbf{J}_e}{\varepsilon_0 \varepsilon} \right), \quad (6)$$

where the terms $\mathbf{J}_e = \mathbf{J}_f - i\omega \delta \mathbf{P}$ and $\mathbf{J}_m = -i\omega \mu_0 \delta \mathbf{M}$ represent the polarization and magnetization noise currents that act as field sources also in the classical fluctuational electrodynamics [27, 28]. The electric term \mathbf{J}_e includes contributions from both the electric currents due to free charges (which amount to zero for insulating dielectrics) as well as polarization terms associated with dipole currents and thermal dipole fluctuations. For the magnetic term \mathbf{J}_m , the only contribution arises from the magnetic dipoles.

For simplicity, we limit the present analysis to the case of normal incidence in a structure where the material parameters only depend on the position coordinate x and formulate Eqs. (5) and (6) as a single polarization scalar problem where the electric and magnetic fields are parallel to the y and z axes, respectively. In the QFED framework, the components of the classical fields and currents

in Eqs. (1)–(6) are replaced by corresponding quantum field operators $\hat{E}^+(x, \omega)$ and $\hat{H}^+(x, \omega)$ and noise current operators $\hat{J}_e^+(x, \omega)$ and $\hat{J}_m^+(x, \omega)$ to account for the quantum features of the field and the noise statistics. From Eqs. (5) and (6) it follows that, in our case of normal incidence, the noise current operators $\hat{J}_e^+(x, \omega)$ and $\hat{J}_m^+(x, \omega)$ describe noise current components that are parallel to the electric and magnetic fields, respectively. In the scalar form, the equations in Eqs. (5) and (6) then simplify to

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{\partial \hat{E}^+(x, \omega)}{\mu_0 \mu(x, \omega) \partial x} \right) + \omega^2 \varepsilon_0 \varepsilon(x, \omega) \hat{E}^+(x, \omega) \\ = -i\omega \hat{J}_e^+(x, \omega) - \frac{\partial}{\partial x} \left(\frac{\hat{J}_m^+(x, \omega)}{\mu_0 \mu(x, \omega)} \right), \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{\partial \hat{H}^+(x, \omega)}{\varepsilon_0 \varepsilon(x, \omega) \partial x} \right) + \omega^2 \mu_0 \mu(x, \omega) \hat{H}^+(x, \omega) \\ = -i\omega \hat{J}_m^+(x, \omega) - \frac{\partial}{\partial x} \left(\frac{\hat{J}_e^+(x, \omega)}{\varepsilon_0 \varepsilon(x, \omega)} \right). \end{aligned} \quad (8)$$

Note that these equations are not independent as either equation allows fully solving the system, leaving the calculation of the remaining fields a simple task when applying the appropriate Maxwell's equation. In the following, we will mainly use Eq. (7) as the starting point for further analysis.

B. Green's functions

In order to write the solution of Eq. (7) in a general form, we first define the electric Green's function $G_{ee}(x, \omega, x')$ that satisfies

$$\frac{\partial}{\partial x} \left(\frac{\partial G_{ee}(x, \omega, x')}{\mu(x, \omega) \partial x} \right) + k_0^2 \varepsilon(x, \omega) G_{ee}(x, \omega, x') = -\delta(x - x'), \quad (9)$$

where $k_0 = \omega/c$ is the wavenumber in vacuum with the vacuum velocity of light c . In terms of the electric Green's function, the solution of Eq. (7) is written as

$$\begin{aligned} \hat{E}^+(x, \omega) \\ = \mu_0 \int_{-\infty}^{\infty} G_{ee}(x, \omega, x') \left[i\omega \hat{J}_e^+(x', \omega) + \frac{\partial}{\partial x'} \left(\frac{\hat{J}_m^+(x', \omega)}{\mu_0 \mu(x', \omega)} \right) \right] dx' \\ = i\omega \mu_0 \int_{-\infty}^{\infty} G_{ee}(x, \omega, x') \hat{J}_e^+(x', \omega) dx' \\ + k_0 \int_{-\infty}^{\infty} G_{em}(x, \omega, x') \hat{J}_m^+(x', \omega) dx', \end{aligned} \quad (10)$$

where, in the case of the second term, we have applied integration by parts with the boundary condition that the Green's functions go to zero at infinities as they are exponentially decaying in lossy media and lossless media can be described in the limit of small losses. We have also defined the exchange Green's function $G_{em}(x, \omega, x')$

as

$$G_{\text{em}}(x, \omega, x') = -\frac{\partial G_{\text{ee}}(x, \omega, x')}{k_0 \mu(x', \omega) \partial x'}. \quad (11)$$

Solving for the magnetic field in the Maxwell's equation in Eq. (3) and substituting the electric field operator in terms of the Green's functions in Eq. (10) gives

$$\begin{aligned} \hat{H}^+(x, \omega) &= \frac{1}{i\omega\mu_0\mu(x, \omega)} \left(\hat{J}_{\text{m}}^+(x, \omega) + \frac{\partial \hat{E}^+(x, \omega)}{\partial x} \right) \\ &= \frac{1}{i\omega\mu_0\mu(x, \omega)} \left(\hat{J}_{\text{m}}^+(x, \omega) + i\omega\mu_0 \int_{-\infty}^{\infty} \frac{\partial G_{\text{ee}}(x, \omega, x')}{\partial x} \right. \\ &\quad \times \hat{J}_{\text{e}}^+(x', \omega) dx' + k_0 \int_{-\infty}^{\infty} \frac{\partial G_{\text{em}}(x, \omega, x')}{\partial x} \hat{J}_{\text{m}}^+(x', \omega) dx' \Big) \\ &= k_0 \int_{-\infty}^{\infty} \frac{\partial G_{\text{ee}}(x, \omega, x')}{k_0 \mu(x, \omega) \partial x} \hat{J}_{\text{e}}^+(x', \omega) dx' \\ &\quad - \frac{ik_0^2}{\omega\mu_0} \int_{-\infty}^{\infty} \left[\frac{\partial G_{\text{em}}(x, \omega, x')}{k_0 \mu(x, \omega) \partial x} + \frac{\delta(x - x')}{k_0^2 \mu(x, \omega)} \right] \hat{J}_{\text{m}}^+(x', \omega) dx' \\ &= k_0 \int_{-\infty}^{\infty} G_{\text{me}}(x, \omega, x') \hat{J}_{\text{e}}^+(x', \omega) dx' \\ &\quad + i\omega\varepsilon_0 \int_{-\infty}^{\infty} G_{\text{mm}}(x, \omega, x') \hat{J}_{\text{m}}^+(x', \omega) dx', \end{aligned} \quad (12)$$

where we have defined the magnetic Green's function $G_{\text{mm}}(x, \omega, x')$ and the exchange Green's function $G_{\text{me}}(x, \omega, x')$ as

$$G_{\text{me}}(x, \omega, x') = \frac{\partial G_{\text{ee}}(x, \omega, x')}{k_0 \mu(x, \omega) \partial x}, \quad (13)$$

$$G_{\text{mm}}(x, \omega, x') = -\frac{\partial G_{\text{em}}(x, \omega, x')}{k_0 \mu(x, \omega) \partial x} - \frac{\delta(x - x')}{k_0^2 \mu(x, \omega)}. \quad (14)$$

By using Eqs. (11) and (14), one also obtains an expression of the magnetic Green's function $G_{\text{mm}}(x, \omega, x')$ directly in terms of the electric Green's function $G_{\text{ee}}(x, \omega, x')$ as

$$G_{\text{mm}}(x, \omega, x') = \frac{\partial^2 G_{\text{ee}}(x, \omega, x')}{k_0^2 \mu(x, \omega) \mu(x', \omega) \partial x \partial x'} - \frac{\delta(x - x')}{k_0^2 \mu(x, \omega)}. \quad (15)$$

In Eq. (15), the first term has a discontinuity at $x = x'$ due to the discontinuity of the second order derivative of $G_{\text{ee}}(x, \omega, x')$. However, this discontinuity is completely balanced by the second term rendering $G_{\text{mm}}(x, \omega, x')$ continuous everywhere.

The electric and magnetic Green's functions obey the general reciprocity relations $G_{\text{ee}}(x, \omega, x') = G_{\text{ee}}(x', \omega, x)$ and $G_{\text{mm}}(x, \omega, x') = G_{\text{mm}}(x', \omega, x)$ [27]. The reciprocity relation for the exchange Green's functions $G_{\text{me}}(x, \omega, x') = -G_{\text{em}}(x', \omega, x)$, follows from the definitions in Eqs. (11) and (13) and the reciprocity relations of $G_{\text{ee}}(x, \omega, x')$ and $G_{\text{mm}}(x, \omega, x')$.

The Green's functions depend on the problem geometry via the material permittivity and permeability and they are continuous at material interfaces which follows from the continuity of the electric and magnetic fields $\hat{E}^+(x, \omega)$ and $\hat{H}^+(x, \omega)$. For example, in a homogeneous space the electric and magnetic Green's functions $G_{\text{ee}}(x, \omega, x')$ and $G_{\text{mm}}(x, \omega, x')$ are

$$G_{\text{ee}}(x, \omega, x') = \mu(\omega) \frac{ie^{ik(\omega)|x-x'|}}{2k(\omega)}, \quad (16)$$

$$G_{\text{mm}}(x, \omega, x') = \varepsilon(\omega) \frac{ie^{ik(\omega)|x-x'|}}{2k(\omega)}, \quad (17)$$

where $k(\omega) = k_0 n(\omega)$ is the wavenumber in the medium. A simple method to calculate the Green's functions in more general stratified media is described in Appendix A.

C. Noise operators

In order to determine the forms of the noise current operators $\hat{J}_{\text{e}}^+(x, \omega)$ and $\hat{J}_{\text{m}}^+(x, \omega)$, we require that the resulting electric and magnetic field operators, related to the noise source operators by Eq. (7) and (8), obey the well-known canonical commutation relations, i.e., $[\hat{A}(x, t), \hat{E}(x', t)] = -i\hbar/(\varepsilon_0 S) \delta(x - x')$ [20, 29, 30]. For purely dielectric media, studied in Refs. [15] and [30], the electric noise current operator is directly proportional to a bosonic annihilation operator $\hat{f}(x, \omega)$ satisfying the canonical commutation relation $[\hat{f}(x, \omega), \hat{f}^\dagger(x', \omega')] = \delta(x - x') \delta(\omega - \omega')$ through $\hat{J}^+(x, \omega) = j_0(x, \omega) \hat{f}(x, \omega)$ where $j_0(x, \omega)$ is a normalization factor given by $j_0(x, \omega) = \sqrt{4\pi\hbar\omega^2\varepsilon_0 \text{Im}[n(x, \omega)^2]/S}$, in which S is the area of quantization in the y - z plane and \hbar is the reduced Planck constant. The operator $\hat{f}(x, \omega)$ gives the local source field number operator $\hat{n}(x, \omega) = \int \hat{f}^\dagger(x, \omega) \hat{f}(x', \omega') dx' d\omega'$ whose expectation value $\langle \hat{n}(x, \omega) \rangle$ is given by the Bose-Einstein distribution as $\langle \hat{n}(x, \omega) \rangle = 1/[e^{\hbar\omega/[k_B T(x)]} - 1]$, in which $T(x)$ is the possibly position-dependent temperature profile of the medium [15]. In the formalism for dielectrics, the derivation of the coefficient $j_0(x, \omega)$ assumes the relation $\varepsilon(x, \omega) = n(x, \omega)^2$ [30], which is not satisfied in the case of magnetic media. It also follows that the canonical commutation relations of fields would not be satisfied in the case of magnetic media if we just neglected the magnetic noise current operator and assumed the same form for the electric noise current operator as that in purely dielectric media.

Thus, to preserve the commutation relations and to find the correct form of the current operators, we allow an additional degree of freedom to conform with the addition of the magnetic noise current operator. The simplest possible current operator form using two independent bosonic source field operators $\hat{f}_{\text{e}}(x, \omega)$ and $\hat{f}_{\text{m}}(x, \omega)$ is $\hat{J}_{\text{e}}^+(x, \omega) = j_{0,\text{e}}(x, \omega) \hat{f}_{\text{e}}(x, \omega)$ and $\hat{J}_{\text{m}}(x, \omega) =$

$j_{0,m}(x,\omega)\hat{f}_m(x,\omega)$, where $j_{0,e}(x,\omega)$ and $j_{0,m}(x,\omega)$ are normalization factors. The above forms can also be partly motivated by the fact that the electric and magnetic current operators $\hat{J}_e^+(x,\omega)$ and $\hat{J}_m^+(x,\omega)$ describe currents in different directions. The bosonic source field operators $\hat{f}_e(x,\omega)$ and $\hat{f}_m(x,\omega)$ are assumed to obey the same canonical commutation relation $[\hat{f}_j(x,\omega), \hat{f}_k^\dagger(x',\omega')] = \delta_{jk}\delta(x-x')\delta(\omega-\omega')$, where $j, k \in \{e, m\}$, as above. Similarly, they also define two separate local source field number operators $\hat{\eta}_e(x,\omega)$ and $\hat{\eta}_m(x,\omega)$. In the case of a thermal source field described by the Bose-Einstein distribution, the expectation values $\langle\hat{\eta}_e(x,\omega)\rangle$ and $\langle\hat{\eta}_m(x,\omega)\rangle$ are additionally equal and denoted by $\langle\hat{\eta}(x,\omega)\rangle$. The normalization factors $j_{0,e}(x,\omega)$ and $j_{0,m}(x,\omega)$ can be determined apart from the possible phase factors by requiring that the vector potential and electric field operators obey the canonical commutation relation $[\hat{A}(x,t), \hat{E}(x',t)] = -i\hbar/(\varepsilon_0 S)\delta(x-x')$ [20, 29, 30]. As a result from the calculation presented in Appendix B, we obtain $j_{0,e}(x,\omega) = \sqrt{4\pi\hbar\omega^2\varepsilon_0\varepsilon_i(x,\omega)/S}$ and $j_{0,m}(x,\omega) = \sqrt{4\pi\hbar\omega^2\mu_0\mu_i(x,\omega)/S}$. This essentially proves that neither of the two noise source operators $\hat{f}_e(x,\omega)$ and $\hat{f}_m(x,\omega)$ can be neglected.

D. Field fluctuations, photon numbers, and local densities of states

In the case of purely dielectric media, the formulas for the field fluctuations, photon numbers, Poynting vector, and local densities of states as described in the QFED are presented in Refs. 15 and 16. In the present case, the general form of the equations stays the same but the densities of states are substantially modified. For completeness, we review below the general formulae while presenting the new generalized nonlocal and interference densities of states.

The spectral components of the time domain electric and magnetic field fluctuations and the energy density $\langle\hat{u}(x,t)\rangle_\omega = \frac{1}{2}|\varepsilon_0\varepsilon(x,\omega)|\langle\hat{E}(x,t)^2\rangle_\omega + \frac{1}{2}|\mu_0\mu(x,\omega)|\langle\hat{H}(x,t)^2\rangle_\omega$ for a single polarization and angular frequency ω are written in terms of the photon-number expectation values as [16]

$$\langle\hat{E}(x,t)^2\rangle_\omega = \frac{\hbar\omega}{\varepsilon_0}\rho_e(x,\omega)\left(\langle\hat{n}_e(x,\omega)\rangle + \frac{1}{2}\right), \quad (18)$$

$$\langle\hat{H}(x,t)^2\rangle_\omega = \frac{\hbar\omega}{\mu_0}\rho_m(x,\omega)\left(\langle\hat{n}_m(x,\omega)\rangle + \frac{1}{2}\right), \quad (19)$$

$$\langle\hat{u}(x,t)\rangle_\omega = \hbar\omega\rho_{\text{tot}}(x,\omega)\left(\langle\hat{n}_{\text{tot}}(x,\omega)\rangle + \frac{1}{2}\right). \quad (20)$$

The photon-number expectation values $\langle\hat{n}_j(x,\omega)\rangle$, $j \in \{e, m, \text{tot}\}$, in Eqs. (18)–(20) are given by

$$\langle\hat{n}_j(x,\omega)\rangle = \frac{\int_{-\infty}^{\infty}\rho_{\text{NL},j}(x,\omega,x')\langle\hat{\eta}(x',\omega)\rangle dx'}{\int_{-\infty}^{\infty}\rho_{\text{NL},j}(x,\omega,x')dx'}. \quad (21)$$

In contrast to purely dielectric media, the nonlocal densities of states (NLDOSs) $\rho_{\text{NL},j}(x,\omega,x')$ now include additional terms originating from magnetic field-matter interactions. The generalized NLDOSs are given by

$$\begin{aligned} \rho_{\text{NL},e}(x,\omega,x') &= \frac{2\omega^3}{\pi c^4 S} \left[\varepsilon_i(x',\omega)|G_{ee}(x,\omega,x')|^2 + \mu_i(x',\omega)|G_{em}(x,\omega,x')|^2 \right], \end{aligned} \quad (22)$$

$$\begin{aligned} \rho_{\text{NL},m}(x,\omega,x') &= \frac{2\omega^3}{\pi c^4 S} \left[\varepsilon_i(x',\omega)|G_{me}(x,\omega,x')|^2 + \mu_i(x',\omega)|G_{mm}(x,\omega,x')|^2 \right], \end{aligned} \quad (23)$$

$$\begin{aligned} \rho_{\text{NL,tot}}(x,\omega,x') &= \frac{|\varepsilon(x,\omega)|}{2}\rho_{\text{NL},e}(x,\omega,x') + \frac{|\mu(x,\omega)|}{2}\rho_{\text{NL},m}(x,\omega,x'). \end{aligned} \quad (24)$$

The local densities of states (LDOSs) also existing in the denominator of Eq. (21) are given in terms of the NLDOSs as

$$\rho_j(x,\omega) = \int_{-\infty}^{\infty}\rho_{\text{NL},j}(x,\omega,x')dx'. \quad (25)$$

The electric and magnetic LDOSs in Eq. (25) with $j \in \{e, m\}$ are related to the imaginary parts of the respective Green's functions $G_{ee}(x,\omega,x)$ and $G_{mm}(x,\omega,x)$ as

$$\rho_j(x,\omega) = \frac{2\omega}{\pi c^2 S} \text{Im}[G_{jj}(x,\omega,x)]. \quad (26)$$

Note that the electric and magnetic LDOSs are directly given by the imaginary parts of the Green's functions $G_{ee}(x,\omega,x)$ and $G_{mm}(x,\omega,x)$ even though the NLDOSs in Eqs. (22) and (23) also depend on the Green's functions $G_{em}(x,\omega,x)$ and $G_{me}(x,\omega,x)$. This manifests the intimate coupling of the four Green's functions. Note that the obtained LDOSs are equivalent to those obtained by using the conventional fluctuational electrodynamics in the case of normal incidence [26, 31, 32].

The position-dependent photon-ladder operators contributing to the effective photon-number expectation values in Eq. (21) can be obtained by the same procedure as that presented for dielectric media in Ref. 15. The only exception is that we now have two commuting source field operators $\hat{f}_e(x,\omega)$ and $\hat{f}_m(x,\omega)$ instead of a single operator $\hat{f}(x,\omega)$. The resulting expression for the ladder operators reads

$$\begin{aligned} \hat{a}_j(x,\omega) &= \frac{1}{\sqrt{\rho_j(x,\omega)}} \int_{-\infty}^{\infty} \left[\sqrt{\rho_{\text{NL},j,e}(x,\omega,x')} \hat{f}_e(x',\omega) \right. \\ &\quad \left. + \sqrt{\rho_{\text{NL},j,m}(x,\omega,x')} \hat{f}_m(x',\omega) \right] dx', \end{aligned} \quad (27)$$

where $\rho_{\text{NL},j,e}(x,\omega,x')$ and $\rho_{\text{NL},j,m}(x,\omega,x')$ with $j \in \{e, m\}$ denote, respectively, the first and the second terms of Eqs. (22) and (23). The total NLDOS terms

$\rho_{\text{NL,tot,e}}(x, \omega, x')$ and $\rho_{\text{NL,tot,m}}(x, \omega, x')$ are calculated by using Eq. (24) with the corresponding terms in the electric and magnetic NLDOSs.

As in the case of dielectric media [15, 17], we apply the general definition of the quantum optical Poynting vector operator as a normal ordered operator in terms of the positive and negative frequency parts of the electric and magnetic field operators, given by $\hat{S}(x, t) =: \hat{E}(x, t)\hat{H}(x, t) := \hat{E}^-(x, t)\hat{H}^+(x, t) + \hat{H}^-(x, t)\hat{E}^+(x, t)$ [33]. Substituting the electric and magnetic field operators in Eqs. (10) and (12) into the Poynting vector definition and taking the expectation value results in

$$\langle \hat{S}(x, t) \rangle_\omega = \hbar\omega v(x, \omega) \int_{-\infty}^{\infty} \rho_{\text{IF}}(x, \omega, x') \langle \hat{\eta}(x', \omega) \rangle dx', \quad (28)$$

where $v(x, \omega) = c/n_r(x, \omega)$ is the energy propagation velocity, $n_r(x, \omega)$ is the real part of the refractive index, and the quantity $\rho_{\text{IF}}(x, \omega, x')$, introduced for nonmagnetic media in Ref. 17, is referred to as the interference density of states (IFDOS) and it is, in the present case, given by

$$\begin{aligned} \rho_{\text{IF}}(x, \omega, x') &= \frac{2\omega^2 n_r(x, \omega)}{\pi c^4 S} \left[\varepsilon_i(x', \omega) \text{Re} \left(i\omega G_{\text{ee}}(x, \omega, x') G_{\text{me}}^*(x, \omega, x') \right) \right. \\ &\quad \left. + \mu_i(x', \omega) \text{Re} \left(i\omega G_{\text{mm}}(x, \omega, x') G_{\text{em}}^*(x, \omega, x') \right) \right]. \end{aligned} \quad (29)$$

The integral of the IFDOS with respect to x' is always zero as required, e.g. by the fact that in a medium in thermal equilibrium, there is no net energy flow [17]. In addition, it is important to note that the total Poynting vector expectation value in Eq. (28) is always continuous at interfaces, which is necessary due to the conservation of energy. Note that, in the QFED framework, it is also possible to apply the density of states concepts to express the Poynting vector expectation value in Eq. (28) in terms of the left and right propagating photon-number expectation values by using the procedure described in Ref. 17.

E. Macroscopic emission and absorption operators and thermal balance

A particularly insightful view of the effective photon numbers is provided by their connection to local thermal balance between the field and matter studied in the case of nonmagnetic media in Ref. 15. Here we present the corresponding thermal balance equation in the case of general media including field-matter interactions through the magnetic field. First, we present the emission and absorption operators $\hat{Q}_{\text{em}}(x, t)$ and $\hat{Q}_{\text{abs}}(x, t)$ that capture the macroscopic nature of the material layers that are assumed to act as constant memoryless reservoirs. In terms of the field and the noise current operators, the normal

ordered emission and absorption operators $\hat{Q}_{\text{em}}(x, t)$ and $\hat{Q}_{\text{abs}}(x, t)$ are given by

$$\hat{Q}_{\text{em}}(x, t) = - : \hat{J}_e(x, t) \hat{E}(x, t) : - : \hat{J}_m(x, t) \hat{H}(x, t) :, \quad (30)$$

$$\hat{Q}_{\text{abs}}(x, t) = : \hat{J}_{e,\text{abs}}(x, t) \hat{E}(x, t) : + : \hat{J}_{m,\text{abs}}(x, t) \hat{H}(x, t) :. \quad (31)$$

The second terms describe the field-matter interactions through the magnetic field and they are not present in the formalism for dielectrics. Whereas the emission current operators $\hat{J}_e(x, t)$ and $\hat{J}_m(x, t)$ directly present the field sources as shown in frequency domain in Eq. (7) and (8), the absorption current operators $\hat{J}_{e,\text{abs}}(x, t)$ and $\hat{J}_{m,\text{abs}}(x, t)$ describe secondary currents that are induced by the electric and magnetic fields. Therefore, the operators $\hat{J}_e(x, t)$ and $\hat{J}_m(x, t)$ can be seen to correspond to the classical free current densities whereas the operators $\hat{J}_{e,\text{abs}}(x, t)$ and $\hat{J}_{m,\text{abs}}(x, t)$ correspond to the classical bound current densities [24]. The operators $\hat{J}_{e,\text{abs}}(x, t)$ and $\hat{J}_{m,\text{abs}}(x, t)$ are written in the spectral domain as $\hat{J}_{e,\text{abs}}^+(x, \omega) = -i\omega\varepsilon_0\chi_e(x, \omega)\hat{E}^+(x, \omega)$ and $\hat{J}_{m,\text{abs}}^+(x, \omega) = -i\omega\mu_0\chi_m(x, \omega)\hat{H}^+(x, \omega)$, where $\chi_e(x, \omega) = \varepsilon(x, \omega) - 1$ and $\chi_m(x, \omega) = \mu(x, \omega) - 1$ are the electric and magnetic susceptibilities of the medium. The total current density operators are then given by $\hat{J}_{e,\text{tot}}(x, t) = \hat{J}_e(x, t) + \hat{J}_{e,\text{abs}}(x, t)$ and $\hat{J}_{m,\text{tot}}(x, t) = \hat{J}_m(x, t) + \hat{J}_{m,\text{abs}}(x, t)$.

The net emission operator $\hat{Q}(x, t) = \hat{Q}_{\text{em}}(x, t) - \hat{Q}_{\text{abs}}(x, t)$, whose expectation value equals the divergence of the Poynting vector expectation value in Eq. (28) [15], is given in terms of the electric and magnetic field operators and the total current density operators as $\hat{Q}(x, t) =: \hat{J}_{e,\text{tot}}(x, t) \hat{E}(x, t) : + : \hat{J}_{m,\text{tot}}(x, t) \hat{H}(x, t) :.$ In the case of dielectrics, the spectral component of the expectation value of the net emission operator can be written in terms of the electric LDOS and the effective electric field photon number expectation value [15]. In the present case, we obtain the net emission expectation value in terms of both the electric and magnetic LDOSs and the effective electric and magnetic field photon numbers as

$$\begin{aligned} \langle \hat{Q}(x, t) \rangle_\omega &= \hbar\omega^2 \varepsilon_i(x, \omega) \rho_e(x, \omega) [\langle \hat{\eta}(x, \omega) \rangle - \langle \hat{n}_e(x, \omega) \rangle] \\ &\quad + \hbar\omega^2 \mu_i(x, \omega) \rho_m(x, \omega) [\langle \hat{\eta}(x, \omega) \rangle - \langle \hat{n}_m(x, \omega) \rangle]. \end{aligned} \quad (32)$$

At global thermal equilibrium, the effective electric and magnetic photon-number expectation values $\langle \hat{n}_e(x, \omega) \rangle$ and $\langle \hat{n}_m(x, \omega) \rangle$ both reach the source field value $\langle \hat{\eta}(x, \omega) \rangle = \langle \hat{\eta}_0 \rangle$ when the net emission rate in Eq. (32) becomes zero. In resonant systems where the energy exchange is dominated by a narrow frequency band, condition $\langle \hat{Q}(x, t) \rangle_\omega = 0$ can be used to determine the approximate steady state temperature of a weakly interacting resonant particle [34].

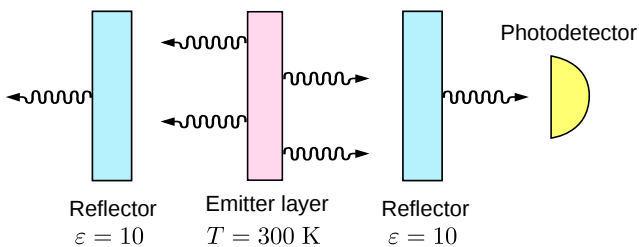


FIG. 1. (Color online) Schematic illustration of the setup for measuring the emissivity of electric and magnetic emitter layers placed in the middle of the vacuum cavity formed by two reflectors. The electric and magnetic LDOSs are position-dependent inside the cavity which results in significantly different emissivities of electric and magnetic emitters as detected by a photodetector outside the cavity.

III. EXAMPLES

To investigate the physical implications and advantages of the concepts presented in Sec. II we apply the methods to study the emissivities of electric and magnetic emitters placed in the middle of a vacuum cavity to show that it is possible to directly probe and demonstrate the essentially independent nature of the electric and magnetic source terms.

In the example, a thin heated material layer which interacts with the electromagnetic field through electric or magnetic interaction terms is placed in the middle of a $10\text{ }\mu\text{m}$ thick vacuum cavity as illustrated in Fig. 1. The relative permittivity and permeability of the lossless $1\text{ }\mu\text{m}$ thick cavity walls are $\varepsilon = 10$ and $\mu = 1$, resulting in the cavity wall power reflection coefficient $R = 0.64$ for the second cavity resonance with energy $\hbar\omega = 0.119\text{ eV}$ ($\lambda = 10.4\text{ }\mu\text{m}$). We focus on the second resonance since it exhibits a node for the electric field and anti-node for the magnetic field in the middle of the cavity.

Figure 2(a) shows the electric and magnetic LDOSs for the second resonant photon energy. Inside the cavity, the electric LDOS has two maxima and the magnetic LDOS, respectively, has two minima. In the middle of the cavity, the electric LDOS obtains its minimum value which is close to zero. The magnetic LDOS instead obtains its maximum value at the same point. The LDOSs determine the local field-matter interactions as seen in the net emission rate in Eq. (32).

Figure 2(b) shows the effective photon number of the total electromagnetic field for the second resonant energy when a $1\text{ }\mu\text{m}$ thick electric or magnetic emitter layer with temperature $T = 300\text{ K}$ is placed in the middle of the cavity. The electric emitter layer has a relative permittivity $\varepsilon = 1.1 + 0.1i$ and a permeability $\mu = 1$ and the magnetic emitter layer, respectively, has a relative permittivity $\varepsilon = 1$ and a permeability $\mu = 1.1 + 0.1i$. The photon number is piecewise constant in all lossless media in the geometry. It can be seen that the magnitude of the photon number is significantly larger in the case of the

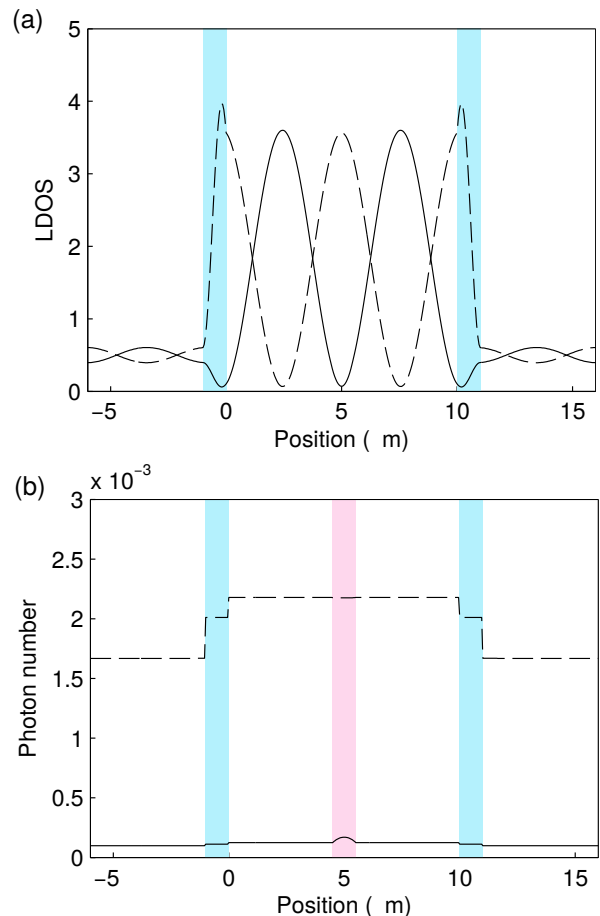


FIG. 2. (Color online) (a) Electric (solid line) and magnetic (dashed line) LDOSs in the vicinity of a vacuum cavity with relative wall permittivity $\varepsilon = 10$ and permeability $\mu = 1$ for the second resonant energy $\hbar\omega = 0.119\text{ eV}$ ($\lambda = 10.4\text{ }\mu\text{m}$). (b) Effective photon number of the total electromagnetic field for the above photon energy in the case of electric (solid line) and magnetic (dashed line) emitter layers at temperature $T = 300\text{ K}$ placed in the middle of the cavity. The electric emitter layer has relative permittivity $\varepsilon = 1.1 + 0.1i$ and permeability $\mu = 1$ and the magnetic emitter layer has relative permittivity $\varepsilon = 1$ and permeability $\mu = 1.1 + 0.1i$. The width of the cavity is $10\text{ }\mu\text{m}$ and the thickness of the cavity walls and emitter layers is $1\text{ }\mu\text{m}$. The LDOSs are given in the units of $2/(\pi cS)$.

magnetic emitter layer due to the different emissivities following from the electric field node and the magnetic field anti-node at the position of the emitter layer. This behavior, which manifests the magnetic Purcell effect, should be directly experimentally measurable by detecting the normally emitted field outside the cavity. The experimental verification of the phenomenon would therefore demonstrate the essentially independent nature of the polarization and magnetization source terms. At high frequencies the magnetic emitter layer would possibly need to be a metamaterial as the frequency-dependent permeability of conventional materials becomes unity.

IV. CONCLUSIONS

In conclusion, we have formulated a generalized QFED noise operator formalism that is able to consistently describe the effective photon number, field-matter interactions, and the formation of thermal balance in nonequilibrium conditions in general isotropic media. It was shown that two commuting bosonic noise operators are needed to describe the field sources for a single polarization in order to maintain the well-known canonical commutation relations for the field operators. The two bosonic noise operators are directly related to the electric and magnetic field-matter interactions in the medium.

We have also used the model to predict how thermal emission from electric or magnetic emitters changes in a configuration where the emitters are located in an optical cavity. The results suggest that it is possible to design a conceptually straightforward experimental setup to differentiate between the two independent noise sources with fundamentally different origin.

ACKNOWLEDGMENTS

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Appendix A: Green's functions

1. Multi-interface reflection and transmission coefficients

We first define the conventional single interface electric and magnetic field reflection and transmission coefficients r_e , r_m , t_e , t_m . The materials on the left and right have relative permittivities and permeabilities ε_1 , μ_1 , ε_2 , and μ_2 , and refractive indices $n_1 = \sqrt{\varepsilon_1 \mu_1}$ and $n_2 = \sqrt{\varepsilon_2 \mu_2}$. For left normal incidence the reflection and transmission coefficients are given by

$$\begin{aligned} r_e &= \frac{\mu_2 n_1 - \mu_1 n_2}{\mu_2 n_1 + \mu_1 n_2}, & t_e &= \frac{2\mu_2 n_1}{\mu_2 n_1 + \mu_1 n_2}, \\ r_m &= \frac{\varepsilon_2 n_1 - \varepsilon_1 n_2}{\varepsilon_2 n_1 + \varepsilon_1 n_2}, & t_m &= \frac{2\varepsilon_2 n_1}{\varepsilon_2 n_1 + \varepsilon_1 n_2}, \end{aligned} \quad (\text{A1})$$

The reflection and transmission coefficients for right incidence r'_e , r'_m , t'_e , and t'_m are obtained by switching the indices 1 and 2 in the expressions of r_e , r_m , t_e , and t_m .

The multi-interface geometry is defined by interface positions x_l , $l = 1, 2, \dots, N$ separating material layers with relative permittivities and permeabilities ε_l and μ_l , refractive indices n_l and wavenumbers k_l , where $l = 1, 2, \dots, N + 1$. The layer thicknesses are denoted by $d_l = x_l - x_{l-1}$, where $l = 2, \dots, N$. The multi-interface reflection and transmission coefficients $\mathcal{R}_{l,j}$ and $\mathcal{T}_{l,j}$, which account for the multiple reflections in different medium

layers, are recursively given in terms of the single interface reflection and transmission coefficients as

$$\mathcal{R}_{l,j} = \frac{r_{l,j} + \mathcal{R}_{l+1,j} e^{2ik_{l+1}d_{l+1}}}{1 + r_{l,j} \mathcal{R}_{l+1,j} e^{2ik_{l+1}d_{l+1}}} \quad (\text{A2})$$

$$\mathcal{T}_{l,j} = \frac{t_{l,j} \nu_{l+1,j}}{\nu_{l,j} (1 - \mathcal{R}'_{l-1,j} r_{l,j} e^{2ik_l d_l})}, \quad (\text{A3})$$

where $l = 1, 2, \dots, N$, $j \in \{e, m\}$, $\nu_{l,j} = 1/(1 - \mathcal{R}'_{l-1,j} \mathcal{R}_{l,j} e^{2ik_l d_l})$, and $\mathcal{R}'_{0,j} = \mathcal{R}_{N+1,j} = 0$. As in the case of single interface coefficients in Eq. (A1) the primed coefficients denote the coefficients for right incidence. The layers are indexed such that $\mathcal{R}'_{l,j}$ corresponds to the same interface as $\mathcal{R}_{l,j}$. The propagation coefficient for a material layer l of thickness d_l is given by $e^{ik_l d_l}$, the transmission coefficient $\mathcal{T}_{l,l',j}$ from layer l' to layer $l > l' + 1$ is recursively given by $\mathcal{T}_{l,l',j} = \mathcal{T}_{l-1,l',j} \mathcal{T}_{l-1,j} e^{ik_{l-1} d_{l-1}}$ with $\mathcal{T}_{l+1,l',j} = \mathcal{T}'_{l',j}$, and the transmission coefficient $\mathcal{T}'_{l,l',j}$ from layer l' to layer $l < l' - 1$ is given by $\mathcal{T}'_{l,l',j} = \mathcal{T}'_{l+1,l',j} \mathcal{T}'_{l,j} e^{ik_{l+1} d_{l+1}}$ with $\mathcal{T}'_{l-1,l',j} = \mathcal{T}'_{l-1,j}$.

2. Green's functions for layered structures

We write the electric and magnetic Green's functions $G_{ee}(x, \omega, x')$ and $G_{mm}(x, \omega, x')$ for a general layered structure in terms of the scaled Green's functions $\xi_j(x, \omega, x')$ defined below as

$$G_{ee}(x, \omega, x') = \mu(x', \omega) \xi_e(x, \omega, x') \quad (\text{A4})$$

$$G_{mm}(x, \omega, x') = \varepsilon(x', \omega) \xi_m(x, \omega, x'). \quad (\text{A5})$$

In the following, the source point x' is located in layer l' ($x_{l'-1} < x' < x_{l'}$) and field point x is located in layer l ($x_{l-1} < x < x_l$) with $x_0 = -\infty$ and $x_{N+1} = \infty$. In the source layer ($l = l'$), the scaled Green's function has three components as

$$\begin{aligned} \xi_{l=l',j}(x, \omega, x') &= \xi_{0,l'}(x, \omega, x') + \xi_{+,l',j}(x, \omega, x') + \xi_{-,l',j}(x, \omega, x'). \end{aligned} \quad (\text{A6})$$

The component $\xi_{0,l'}(x, \omega, x')$ is the homogeneous space solution and the components $\xi_{+,l',j}(x, \omega, x')$ and $\xi_{-,l',j}(x, \omega, x')$ describe the right and left propagating fields due to the reflections at the interfaces. The homogeneous space solution is given by

$$\xi_{0,l'}(x, \omega, x') = \frac{i e^{ik_{l'} |x - x'|}}{2k_{l'}} \quad (\text{A7})$$

and the right propagating reflection originating compo-

nent is written as

$$\begin{aligned}
& \xi_{+,l',j}(x, \omega, x') \\
&= e^{ik_{l'}(x-x_{l'-1})} \xi_{0,l'}(x_{l'-1}, \omega, x') \mathcal{R}'_{l'-1,j} \\
& \quad \times \sum_{m=0}^{\infty} (\mathcal{R}'_{l'-1,j} \mathcal{R}_{l',j} e^{2ik_{l'}d_{l'}})^m \\
& \quad + e^{ik_{l'}(x-x_{l'-1})} \xi_{0,l'}(x_{l'}, \omega, x') \mathcal{R}'_{l'-1,j} \mathcal{R}_{l',j} e^{ik_j d_j} \\
& \quad \times \sum_{m=0}^{\infty} (\mathcal{R}'_{l'-1,j} \mathcal{R}_{l',j} e^{2ik_{l'}d_{l'}})^m \\
&= e^{ik_{l'}(x-x_{l-1})} \frac{ie^{ik_{l'}(x'-x_{l'-1})}}{2k_{l'}} \nu_{l',j} \mathcal{R}'_{l'-1,j} \\
& \quad + e^{ik_{l'}(x-x_{l'-1})} \frac{ie^{ik_{l'}(x_{l'}-x')}}{2k_{l'}} e^{ik_{l'}d_{l'}} \nu_{l',j} \mathcal{R}'_{l'-1,j} \mathcal{R}_{l'} \\
&= \frac{i}{2k_{l'}} \nu_{l',j} \mathcal{R}'_{l'-1,j} (e^{ik_{l'}(x+x'-2x_{l'-1})} + \mathcal{R}_{l',j} e^{ik_{l'}(x-x'+2d_{l'})}). \tag{A8}
\end{aligned}$$

The first term describes the field component incident from the source point to the left and the second term describes the field component incident from the source point to the right. The factor $\nu_{l',j} = 1/(1 - \mathcal{R}'_{l'-1,j} \mathcal{R}_{l',j} e^{2ik_{l'}d_{l'}})$ arises from the series accounting for the multiple reflections inside the source layer. Respectively, the left propagating reflection originating component is written as

$$\begin{aligned}
& \xi_{-,l',j}(x, \omega, x') \\
&= e^{-ik_{l'}(x-x_{l'})} \xi_{0,l'}(x_{l'}, \omega, x') \mathcal{R}_{l',j} \\
& \quad \sum_{m=0}^{\infty} (\mathcal{R}'_{l'-1,j} \mathcal{R}_{l',j} e^{2ik_{l'}d_{l'}})^m \\
& \quad + e^{-ik_{l'}(x-x_{l'})} \xi_{0,l'}(x_{l'-1}, \omega, x') \mathcal{R}'_{l'-1,j} \mathcal{R}_{l',j} e^{ik_{l'}d_{l'}} \\
& \quad \sum_{m=0}^{\infty} (\mathcal{R}'_{l'-1,j} \mathcal{R}_{l',j} e^{2ik_{l'}d_{l'}})^m \\
&= e^{-ik_{l'}(x-x_{l'})} \frac{ie^{ik_{l'}(x_{l'}-x')}}{2k_{l'}} \nu_{l',j} \mathcal{R}_{l',j} \\
& \quad + e^{-ik_{l'}(x-x_{l'})} \frac{ie^{ik_{l'}(x'-x_{l'-1})}}{2k_{l'}} e^{ik_{l'}d_{l'}} \nu_{l',j} \mathcal{R}'_{l'-1,j} \mathcal{R}_{l',j} \\
&= \frac{i}{2k_{l'}} \nu_{l',j} \mathcal{R}_{l',j} (e^{-ik_{l'}(x+x'-2x_{l'})} + \mathcal{R}'_{l'-1,j} e^{-ik_{l'}(x-x'-2d_{l'})}). \tag{A9}
\end{aligned}$$

Therefore, the total scaled Green's function is given in the source layer by

$$\begin{aligned}
& \xi_{l=l',j}(x, \omega, x') \\
&= \frac{i}{2k_{l'}} \left(e^{ik_{l'}|x-x'|} + \nu_{l',j} \mathcal{R}_{l',j} [e^{-ik_{l'}(x+x'-2x_{l'})} \right. \\
& \quad + \mathcal{R}'_{l'-1,j} e^{-ik_{l'}(x-x'-2d_{l'})}] + \nu_{l',j} \mathcal{R}'_{l'-1,j} \\
& \quad \times [e^{ik_{l'}(x+x'-2x_{l'-1})} + \mathcal{R}_{l',j} e^{ik_{l'}(x-x'+2d_{l'})}] \Big). \tag{A10}
\end{aligned}$$

Writing the scaled Green's functions in other layers is even more straightforward and, as a result, the scaled Green's functions are given in the cases $l > l'$ and $l < l'$

by

$$\begin{aligned}
& \xi_{l>l',j}(x, \omega, x') \\
&= \frac{i}{2k_{l'}} \mathcal{T}_{l,l',j} \left(e^{ik_{l'}(x_{l'}-x')} + \nu_{l',j} \mathcal{R}'_{l'-1,j} [e^{ik_{l'}(x'-x_{l'-1}+d_{l'})} \right. \\
& \quad + \mathcal{R}_{l',j} e^{ik_{l'}(2d_{l'}-x'+x_{l'})}] \Big) \\
& \quad \times \left(e^{ik_{l'}(x-x_{l-1})} + \mathcal{R}_{l,j} e^{-ik_{l'}(x-x_{l-1}-2d_{l'})} \right), \tag{A11}
\end{aligned}$$

$$\begin{aligned}
& \xi_{l<l',j}(x, \omega, x') \\
&= \frac{i}{2k_{l'}} \mathcal{T}_{l,l',j} \left(e^{ik_{l'}(x'-x_{l'-1})} + \nu_{l',j} \mathcal{R}_{l',j} [e^{-ik_{l'}(x'-x_{l'-1}-2d_{l'})} \right. \\
& \quad + \mathcal{R}'_{l'-1,j} e^{ik_{l'}(x'-x_{l'-1}+2d_{l'})}] \Big) \\
& \quad \times \left(e^{-ik_{l'}(x-x_{l-1})} + \mathcal{R}'_{l-1,j} e^{ik_{l'}(x-x_{l-1}+d_{l'})} \right). \tag{A12}
\end{aligned}$$

Appendix B: Canonical commutation relations of fields

Here we determine the normalization factors $j_{0,e}(x, \omega)$ and $j_{0,m}(x, \omega)$ of the noise current operators by requiring that the vector potential and electric field operators obey the canonical commutation relation $[\hat{A}(x, t), \hat{E}(x', t)] = -i\hbar/(\epsilon_0 S) \delta(x - x')$ [20, 29, 30]. The vector potential and electric field operators are given in the frequency domain by

$$\begin{aligned}
& \hat{A}^+(x, \omega) \\
&= \mu_0 \int_{-\infty}^{\infty} j_{0,e}(x', \omega) G_{ee}(x, \omega, x') \hat{f}_e(x', \omega) dx' \\
& \quad + \frac{1}{ic} \int_{-\infty}^{\infty} j_{0,m}(x', \omega) G_{em}(x, \omega, x') \hat{f}_m(x', \omega) dx'. \tag{B1}
\end{aligned}$$

$$\begin{aligned}
& \hat{E}^+(x, \omega) \\
&= i\omega\mu_0 \int_{-\infty}^{\infty} j_{0,e}(x', \omega) G_{ee}(x, \omega, x') \hat{f}_e(x', \omega) dx' \\
& \quad + k_0 \int_{-\infty}^{\infty} j_{0,m}(x', \omega) G_{em}(x, \omega, x') \hat{f}_m(x', \omega) dx'. \tag{B2}
\end{aligned}$$

The frequency domain commutator is obtained by using the frequency domain vector potential and electric field

operators as

$$\begin{aligned}
& [\hat{A}^{+\dagger}(x, \omega), \hat{E}^+(x', \omega')] \\
&= i\omega\mu_0^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} j_{0,e}(y, \omega) j_{0,e}^*(y', \omega') G_{ee}(x, \omega, y) \\
&\quad \times G_{ee}^*(x', \omega', y') [\hat{f}_e^\dagger(y, \omega), \hat{f}_e(y', \omega')] dy dy' \\
&\quad - \frac{k_0}{ic} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} j_{0,m}(y, \omega) j_{0,m}^*(y', \omega') G_{em}(x, \omega, y) \\
&\quad \times G_{em}^*(x', \omega', y') [\hat{f}_m^\dagger(y, \omega), \hat{f}_m(y', \omega')] dy dy' \\
&= \frac{\delta(\omega - \omega')}{i\omega} \left[\omega^2 \mu_0^2 \int_{-\infty}^{\infty} |j_{0,e}(y, \omega)|^2 G_{ee}(x, \omega, y) G_{ee}^*(x', \omega, y) dy \right. \\
&\quad \left. + k_0^2 \int_{-\infty}^{\infty} |j_{0,m}(y, \omega)|^2 G_{em}(x, \omega, y) G_{em}^*(x', \omega, y) dy \right] \\
&= -i \frac{4\pi\hbar\omega}{\varepsilon_0 c^2 S} \delta(\omega - \omega') \left[k_0^2 \int_{-\infty}^{\infty} [\varepsilon_i(y, \omega) + C_1(y, \omega)] \right. \\
&\quad \times G_{ee}(x, \omega, y) G_{ee}^*(x', \omega, y) dy \\
&\quad \left. + k_0^2 \int_{-\infty}^{\infty} [\mu_i(y, \omega) + C_2(y, \omega)] G_{em}(x, \omega, y) G_{em}^*(x', \omega, y) dy \right] \\
&= -i \frac{4\pi\hbar\omega}{\varepsilon_0 c^2 S} \delta(\omega - \omega') \text{Im}[G_{ee}(x, \omega, x')] \\
&\quad - i \frac{4\pi\hbar\omega^3}{\varepsilon_0 c^4 S} \delta(\omega - \omega') \int_{-\infty}^{\infty} C(x, x', y, \omega) dy, \tag{B3}
\end{aligned}$$

where we have first substituted the field operators in Eqs. (B1) and (B2), then applied the commutation relations of operators $\hat{f}_e(y, \omega)$ and $\hat{f}_m(y, \omega)$ after which we have substituted $|j_{0,e}(y, \omega)|^2 = 4\pi\hbar\omega^2\varepsilon_0[\varepsilon_i(y, \omega) + C_1(y, \omega)]/S$ and $|j_{0,m}(y, \omega)|^2 = 4\pi\hbar\omega^2\mu_0[\mu_i(y, \omega) + C_2(y, \omega)]/S$, where $C_1(y, \omega)$ and $C_2(y, \omega)$ are functions that will be determined at the end of the calculation. These substitutions essentially just transform the undetermined factors $|j_{0,e}(y, \omega)|^2$ and $|j_{0,m}(y, \omega)|^2$ to undetermined factors $C_1(y, \omega)$ and $C_2(y, \omega)$. The forms of the substitutions are chosen so that the functions $C_1(y, \omega)$ and $C_2(y, \omega)$ can be shown to be zero at the end of the calculation. In the final step in Eq. (B3), we have applied the Green's function integral identity in Eq. (C6) and denoted

$$\begin{aligned}
C(x, x', y, \omega) &= C_1(y, \omega) G_{ee}(x, \omega, y) G_{ee}^*(x', \omega, y) \\
&\quad + C_2(y, \omega) G_{em}(x, \omega, y) G_{em}^*(x', \omega, y). \tag{B4}
\end{aligned}$$

Next, we present the time domain equal-time commutator. The time domain operators are given in terms of the frequency domain operators as

$$\hat{A}(x, t) = \frac{1}{2\pi} \int_0^\infty \hat{A}^+(x, \omega) e^{-i\omega t} d\omega + H.c., \tag{B5}$$

$$\hat{E}(x, t) = \frac{1}{2\pi} \int_0^\infty \hat{E}^+(x, \omega) e^{-i\omega t} d\omega + H.c., \tag{B6}$$

where $H.c.$ denotes the Hermitian conjugate of the first term. In the calculation of the commutator, we also use the general relation $[\hat{A}^+(x, \omega), \hat{E}^{+\dagger}(x', \omega')] = [\hat{A}^{+\dagger}(x, \omega), \hat{E}^+(x', \omega')]$, which relates the commutators

of the conjugate terms. By using Eq. (B3), the time domain equal-time commutator is then given by

$$\begin{aligned}
& [\hat{A}(x, t), \hat{E}(x', t)] \\
&= \frac{1}{4\pi^2} \int_0^\infty \int_0^\infty [\hat{A}^{+\dagger}(x, \omega), \hat{E}^+(x', \omega')] e^{-i(\omega - \omega')t} d\omega d\omega' \\
&\quad + \frac{1}{4\pi^2} \int_0^\infty \int_0^\infty [\hat{A}^+(x, \omega), \hat{E}^{+\dagger}(x', \omega')] e^{i(\omega - \omega')t} d\omega d\omega' \\
&= -i \frac{2\hbar}{\pi\varepsilon_0 c^2 S} \int_0^\infty \omega \text{Im}[G_{ee}(x, \omega, x')] d\omega \\
&\quad - i \frac{2\hbar}{\pi\varepsilon_0 c^4 S} \int_0^\infty \int_{-\infty}^\infty \omega^3 C(x, x', y, \omega) dy d\omega \\
&= -\frac{\hbar}{\pi\varepsilon_0 c^2 S} \int_0^\infty \omega [G_{ee}(x, \omega, x') - G_{ee}^*(x, \omega, x')] d\omega \\
&\quad - i \frac{2\hbar}{\pi\varepsilon_0 c^4 S} \int_0^\infty \int_{-\infty}^\infty \omega^3 C(x, x', y, \omega) dy d\omega \\
&= -\frac{\hbar}{\pi\varepsilon_0 c^2 S} \int_{-\infty}^\infty \omega G_{ee}(x, \omega, x') d\omega \\
&\quad - i \frac{2\hbar}{\pi\varepsilon_0 c^4 S} \int_0^\infty \int_{-\infty}^\infty \omega^3 C(x, x', y, \omega) dy d\omega. \tag{B7}
\end{aligned}$$

where we have used the relation $2i\text{Im}(z) = z - z^*$, $z \in \mathbb{C}$, and then, for the resulting conjugate term, the Green's function conjugation relation $G_{ee}^*(x, \omega, x') = G_{ee}(x, -\omega, x')$ and the change of variables $\omega \rightarrow -\omega$. This allows us to express the first integral term as an integral over the whole real axis.

By substituting different Green's function terms into the first term in the result of Eq. (B7), it can be shown that only the homogeneous space solution term of the Green's function contributes to the result. Substituting the homogeneous space solution term of the Green's function into the first term in Eq. (B7), we obtain

$$\begin{aligned}
& [\hat{A}(x, t), \hat{E}(x', t)] \\
&= -\frac{i\hbar}{\pi\varepsilon_0 c^2 S} \int_{-\infty}^\infty \omega \mu(x', \omega) \frac{e^{i\omega n(x', \omega)|x-x'|/c}}{2\omega n(x', \omega)/c} d\omega \\
&\quad - i \frac{2\hbar}{\pi\varepsilon_0 c^4 S} \int_0^\infty \int_{-\infty}^\infty \omega^3 C(x, x', y, \omega) dy d\omega \\
&= -\frac{\hbar}{2\pi^2 \varepsilon_0 c^2 S} \int_{-\infty}^\infty \omega \mu(x', \omega) \int_{-\infty}^\infty \frac{e^{ik(x-x')}}{k^2 - \omega^2 n(x', \omega)^2/c^2} dk d\omega \\
&= -\frac{\hbar}{2\pi^2 \varepsilon_0 S} \int_{-\infty}^\infty e^{ik(x-x')} \int_{-\infty}^\infty \frac{\omega \mu(x', \omega)}{k^2 c^2 - \omega^2 n(x', \omega)^2} d\omega dk \\
&\quad - i \frac{2\hbar}{\pi\varepsilon_0 c^4 S} \int_0^\infty \int_{-\infty}^\infty \omega^3 C(x, x', y, \omega) dy d\omega \\
&= -\frac{i\hbar}{\varepsilon_0 S} \frac{1}{2\pi} \int_{-\infty}^\infty e^{ik(x-x')} dk \\
&\quad - i \frac{2\hbar}{\pi\varepsilon_0 c^4 S} \int_0^\infty \int_{-\infty}^\infty \omega^3 C(x, x', y, \omega) dy d\omega \\
&= -\frac{i\hbar}{\varepsilon_0 S} \delta(x-x') - i \frac{2\hbar}{\pi\varepsilon_0 c^4 S} \int_0^\infty \int_{-\infty}^\infty \omega^3 C(x, x', y, \omega) dy d\omega, \tag{B8}
\end{aligned}$$

where we have applied the mathematical integral identities in Eqs. (C11) and (C14) and the definition of the Dirac delta function. From the final result, it follows that the second term must be zero as the canonical commutation relation is known to be given by $[\hat{A}(x, t), \hat{E}(x', t)] = -i\hbar/(\varepsilon_0 S)\delta(x - x')$ [20, 29, 30]. Therefore, as the integral of $C(x, x', y, \omega)$ must give zero for all values of x and x' and as the position-dependence of $C(x, x', y, \omega)$ comes directly from two linearly independent Green's functions as presented in Eq. (B4), we must have $C_1(y, \omega) = C_2(y, \omega) = 0$. This condition then fixes the values of the normalization factors $j_{0,e}(x, \omega)$ and $j_{0,m}(x, \omega)$ to

$$j_{0,e}(x, \omega) = \sqrt{4\pi\hbar\omega^2\varepsilon_0\varepsilon_i(x, \omega)/S}, \quad (\text{B9})$$

$$j_{0,m}(x, \omega) = \sqrt{4\pi\hbar\omega^2\mu_0\mu_i(x, \omega)/S}, \quad (\text{B10})$$

which are unique apart from the possible phase factors.

Appendix C: Mathematical identities

1. Green's function integral identities

Here we derive the integral identities for the Green's function used in the final step in the evaluation of Eq. (B3). The electric Green's function obeys the differential equation in Eq. (9) which we write by renaming the variables as

$$\frac{\partial}{\partial y} \left(\frac{\partial G_{ee}(y, \omega, x)}{\mu(y, \omega) \partial y} \right) + k_0^2 \varepsilon(y, \omega) G_{ee}(y, \omega, x) = -\delta(y - x). \quad (\text{C1})$$

We multiply this with the conjugated Green's function $G_{ee}^*(y, \omega, x')$ and integrate over y to obtain

$$\begin{aligned} & \int_{-\infty}^{\infty} G_{ee}^*(y, \omega, x') \frac{\partial}{\partial y} \left(\frac{\partial G_{ee}(y, \omega, x)}{\mu(y, \omega) \partial y} \right) dy \\ & + k_0^2 \int_{-\infty}^{\infty} \varepsilon(y, \omega) G_{ee}^*(y, \omega, x') G_{ee}(y, \omega, x) dy \\ & = -G_{ee}^*(x, \omega, x'). \end{aligned} \quad (\text{C2})$$

Here, the first term can be integrated by parts accounting for the fact that the boundary term becomes zero as the Green's function is exponentially decaying in lossy media and lossless media can be studied in the limit of small losses. Therefore, we get

$$\begin{aligned} & - \int_{-\infty}^{\infty} \mu^*(y, \omega) \frac{\partial G_{ee}(y, \omega, x)}{\mu(y, \omega) \partial y} \frac{\partial G_{ee}^*(y, \omega, x')}{\mu^*(y, \omega) \partial y} dy \\ & + k_0^2 \int_{-\infty}^{\infty} \varepsilon(y, \omega) G_{ee}(y, \omega, x) G_{ee}^*(y, \omega, x') dy \\ & = -G_{ee}^*(x, \omega, x'). \end{aligned} \quad (\text{C3})$$

The first term can be expressed in terms of the Green's function $G_{me}(x, \omega, x')$ by using Eq. (13). Thus, we obtain

$$\begin{aligned} & -k_0^2 \int_{-\infty}^{\infty} \mu^*(y, \omega) G_{me}(y, \omega, x) G_{me}^*(y, \omega, x') dy \\ & + k_0^2 \int_{-\infty}^{\infty} \varepsilon(y, \omega) G_{ee}(y, \omega, x) G_{ee}^*(y, \omega, x') dy \\ & = -G_{ee}^*(x, \omega, x'). \end{aligned} \quad (\text{C4})$$

By applying the Green's function reciprocity relations $G_{ee}(x, \omega, x') = G_{ee}(x', \omega, x)$ and $G_{me}(x, \omega, x') = -G_{em}(x', \omega, x)$, we get

$$\begin{aligned} & -k_0^2 \int_{-\infty}^{\infty} \mu^*(y, \omega) G_{em}(x, \omega, y) G_{em}^*(x', \omega, y) dy \\ & + k_0^2 \int_{-\infty}^{\infty} \varepsilon(y, \omega) G_{ee}(x, \omega, y) G_{ee}^*(x', \omega, y) dy \\ & = -G_{ee}^*(x, \omega, x'). \end{aligned} \quad (\text{C5})$$

Taking the imaginary part and switching the terms gives the final result

$$\begin{aligned} & k_0^2 \int_{-\infty}^{\infty} \varepsilon_i(y, \omega) G_{ee}(x, \omega, y) G_{ee}^*(x', \omega, y) dy \\ & + k_0^2 \int_{-\infty}^{\infty} \mu_i(y, \omega) G_{em}(x, \omega, y) G_{em}^*(x', \omega, y) dy \\ & = \text{Im}[G_{ee}(x, \omega, x')]. \end{aligned} \quad (\text{C6})$$

Respectively, for the magnetic Green's function, we obtain

$$\begin{aligned} & k_0^2 \int_{-\infty}^{\infty} \mu_i(y, \omega) G_{mm}(x, \omega, y) G_{mm}^*(x', \omega, y) dy \\ & + k_0^2 \int_{-\infty}^{\infty} \varepsilon_i(y, \omega) G_{me}(x, \omega, y) G_{me}^*(x', \omega, y) dy \\ & = \text{Im}[G_{mm}(x, \omega, x')]. \end{aligned} \quad (\text{C7})$$

2. Integral identity for k integration

Here we derive the integral identity for k integration used in one of the intermediate steps in Eq. (B8). The integrand in

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{ik(x-x')}}{k^2 - \omega^2 n^2 / c^2} dk \quad (\text{C8})$$

has two poles at positions $k = \pm \omega n / c$. The pole with positive sign is located in the upper half of the complex k plane and the pole with negative sign is located in the lower half plane. When $x > x'$ the integrand goes to zero in the upper half plane as $k \rightarrow \infty$. Therefore, by using the residue theorem we obtain

$$\begin{aligned} & \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{ik(x-x')}}{k^2 - \omega^2 n^2 / c^2} dk \\ & = \text{Res}_{k=\omega n/c} \frac{e^{ik(x-x')}}{k^2 - \omega^2 n^2 / c^2} = \frac{e^{i\omega n(x-x')/c}}{2\omega n/c}. \end{aligned} \quad (\text{C9})$$

When $x < x'$ the integrand, respectively, goes to zero in the lower half of the complex plane as $k \rightarrow \infty$ and, by applying the residue theorem, we obtain

$$\begin{aligned} & \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{ik(x-x')}}{k^2 - \omega^2 n^2/c^2} dk \\ &= - \text{Res}_{k=-\omega n/c} \frac{e^{ik(x-x')}}{k^2 - \omega^2 n^2/c^2} = \frac{e^{-i\omega n(x-x')/c}}{2\omega n/c}. \end{aligned} \quad (\text{C10})$$

The two equations in Eqs. (C9) and (C10) can be combined to give the final result

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{ik(x-x')}}{k^2 - \omega^2 n^2/c^2} dk = \frac{e^{i\omega n|x-x'|/c}}{2\omega n/c}. \quad (\text{C11})$$

3. Integral identity for ω integration

Here we derive the integral identity for ω integration used in one of the intermediate steps in Eq. (B8). The integrand in

$$\int_{-\infty}^{\infty} \frac{\omega \mu(\omega)}{k^2 c^2 - \omega^2 n(\omega)^2} d\omega \quad (\text{C12})$$

has no poles in the upper half of the complex ω plane. The integral along the real ω axis in Eq. (C12) is therefore the negative of the integral around the semicircle at infinity in the upper half plane, so putting $\omega = \Omega e^{i\varphi}$, we obtain

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{\omega \mu(\omega)}{k^2 c^2 - \omega^2 n(\omega)^2} d\omega \\ &= - \lim_{\Omega \rightarrow \infty} \int_0^\pi \frac{i\mu(\Omega e^{2i\varphi})\Omega^2 e^{2i\varphi}}{k^2 c^2 - \Omega^2 n(\Omega e^{2i\varphi})^2 e^{2i\varphi}} d\varphi \\ &= i \int_0^\pi d\varphi = i\pi \end{aligned} \quad (\text{C13})$$

where we have applied the fact that material parameters are analytic functions of frequency and become unity at high frequencies. Thus, we have

$$\int_{-\infty}^{\infty} \frac{\omega \mu(\omega)}{k^2 c^2 - \omega^2 n(\omega)^2} d\omega = i\pi. \quad (\text{C14})$$

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